Student Workbook

Introduction to Logarithmic Functions

Name: ______________________ Date: ______________ Class: ________

ESSENTIAL QUESTION

If an exponential function asks "what power gives this result?", how does a logarithm answer that question — and why are the two functions perfect inverses of each other?

Lesson Overview

A logarithm is the inverse of an exponential function. The expression $\log_b(x) = y$ asks: "To what power must I raise b to get x?" The answer is y. This single idea unlocks the ability to solve exponential equations, work with pH, decibels, the Richter scale, and much more.

$$\log_b(x) = y \iff b^y = x$$

b	base of the logarithm ($b > 0$, $b \neq 1$)
x	argument of the logarithm (must be > 0)
y	the logarithm itself — this is the exponent

Special Logarithms & Key Properties

- Common log: $\log(x) = \log_{10}(x)$
- Natural log: $\ln(x) = \log_e(x)$
- $\log_b(1) = 0$ (since $b^0 = 1$)
- $\log_b(b) = 1$ (since $b^1 = b$)
- $\log_b(b^n) = n$ — inverse property
- $b^{\log_b x} = x$ — inverse property

Worked Examples

EXAMPLE 1

Convert each to logarithmic form: (a) $2^5 = 32$ (b) $10^3 = 1000$ (c) $5^{-2} = 1/25$

- Rule: $b^y = x \rightarrow \log_b(x) = y$
- (a) $2^5 = 32 \rightarrow \log_2(32) = 5$ (base 2, exponent 5, result 32)
- (b) $10^3 = 1000 \rightarrow \log(1000) = 3$ (base 10 = common log)
- (c) $5^{-2} = 1/25 \rightarrow \log_5(1/25) = -2$ (negative exponent is fine)

Answer: (a) $\log_2(32) = 5$ (b) $\log(1000) = 3$ (c) $\log_5(1/25) = -2$

EXAMPLE 2

Convert each to exponential form: (a) $\log_3(81) = 4$ (b) $\log(0.01) = -2$ (c) $\ln(e^3) = 3$

- Rule: $\log_b(x) = y \rightarrow b^y = x$
- (a) $\log_3(81) = 4 \rightarrow 3^4 = 81$
- (b) $\log(0.01) = -2 \rightarrow 10^{-2} = 0.01$
- (c) $\ln(e^3) = 3 \rightarrow e^3 = e^3$ (natural log uses base e)

Answer: (a) $3^4 = 81$ (b) $10^{-2} = 0.01$ (c) $e^3 = e^3$

EXAMPLE 3

Evaluate without a calculator: (a) $\log_2(64)$ (b) $\log_5(125)$ (c) $\log(10{,}000)$ (d) $\log_4(1)$

- a) $\log_2(64)$: ask "2 to what power = 64?" → $2^6 = 64$ → 6
- b) $\log_5(125)$: ask "5 to what power = 125?" → $5^3 = 125$ → 3
- c) $\log(10{,}000)$: ask "10 to what power = 10,000?" → $10^4 = 10{,}000$ → 4
- d) $\log_4(1)$: ask "4 to what power = 1?" → $4^0 = 1$ → 0
- Key fact: $\log_b(1) = 0$ for any valid base b.

Answer: (a) 6 (b) 3 (c) 4 (d) 0

EXAMPLE 4

Evaluate: (a) $\log_3(3^7)$ (b) $5^{\log_5 12}$ (c) $\ln(e)$ (d) $\log_2(1/8)$

- (a) $\log_3(3^7) = 7$ — inverse property: $\log_b(b^n) = n$
- (b) $5^{\log_5 12} = 12$ — inverse property: $b^{\log_b x} = x$
- (c) $\ln(e) = \log_e(e) = 1$, since $e^1 = e$
- (d) $\log_2(1/8)$: $1/8 = 2^{-3}$, so $\log_2(2^{-3}) = -3$

Answer: (a) 7 (b) 12 (c) 1 (d) -3

EXAMPLE 5

Solve for x: (a) $\log_4(x) = 3$ (b) $\log_x(36) = 2$ (c) $\log_2(x) = -4$

- (a) $\log_4(x) = 3 \rightarrow 4^3 = x \rightarrow x = 64$
- (b) $\log_x(36) = 2 \rightarrow x^2 = 36 \rightarrow x = 6$ (base must be positive, not 1)
- (c) $\log_2(x) = -4 \rightarrow 2^{-4} = x \rightarrow x = 1/16$
- Strategy: convert to exponential form, then solve for the unknown.

Answer: (a) x = 64 (b) x = 6 (c) x = 1/16

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

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Introduction to Logarithmic Functions

1 Convert to logarithmic form: (a) $4^3 = 64$ (b) $e^2 \approx 7.389$ (c) $7^{-1} = 1/7$

Hint: Use the rule $b^y = x \rightarrow \log_b(x) = y$. Identify base, exponent, and result.

2 Convert to exponential form: (a) $\log_6(216) = 3$ (b) $\ln(1) = 0$ (c) $\log(100) = 2$

Hint: Use $\log_b(x) = y \rightarrow b^y = x$. For ln, the base is e; for log, the base is 10.

3 Evaluate without a calculator: (a) $\log_3(27)$ (b) $\log_{10}(0.001)$ (c) $\log_5(5)$ (d) $\log_2(1/4)$

Hint: Ask 'base to what power equals the argument?' Use $b^0=1$, $b^1=b$, and negative exponents for fractions.

4 Use inverse properties to evaluate: (a) $\log_7(7^5)$ (b) $4^{\log_4 9}$ (c) $\ln(e^{-3})$

Hint: Inverse properties: $\log_b(b^n) = n$ and $b^{\log_b x} = x$. Apply directly.

5 Solve for x: (a) $\log_3(x) = 4$ (b) $\log_2(x) = -3$ (c) $\log_x(49) = 2$

Hint: Convert to exponential form first: $\log_b(x) = y \rightarrow b^y = x$.

Key Vocabulary

Logarithm	The exponent to which a base must be raised to produce a given number. $\log_b(x) = y$ means $b^y = x$.
Base of a Logarithm	The number b in $\log_b(x)$. Must satisfy $b > 0$ and $b \neq 1$. Determines the 'scale' of the logarithm.
Common Logarithm	A logarithm with base 10, written $\log(x)$ or $\log_{10}(x)$. Used in pH, decibels, and the Richter scale.
Natural Logarithm	A logarithm with base $e \approx 2.718$, written $\ln(x)$. Fundamental in calculus and continuous growth models.
Argument	The input x in $\log_b(x)$. The argument must always be strictly positive ($x > 0$).
Inverse Functions	Two functions that undo each other. $f(x) = b^x$ and $g(x) = \log_b(x)$ are inverses.
Inverse Properties	$\log_b(b^n) = n$ and $b^{\log_b x} = x$. These follow directly from the definition of inverse functions.
Vertical Asymptote	For $f(x) = \log_b(x)$, the y-axis ($x = 0$) is a vertical asymptote — the graph approaches but never crosses it.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 **Which exponential equation is equivalent to $\log_5(125) = 3$?** Multiple Choice

- **A** $3^5 = 125$
- **B** $5^3 = 125$
- **C** $5^{125} = 3$
- **D** $125^3 = 5$

2 **Evaluate $\log_2(32)$.** Multiple Choice

- **A** 4
- **B** 5
- **C** 6
- **D** 16

3 **What is the value of $\log_9(1)$?** Multiple Choice

- **A** 9
- **B** 1
- **C** 0
- **D** undefined

4 **Evaluate $6^{\log_6 17}$.** Multiple Choice

- **A** 6
- **B** 17
- **C** 1
- **D** $\log_6(17)$

5 **Solve: $\log_3(x) = -2$.** Multiple Choice

- **A** $x = -9$
- **B** $x = 9$
- **C** $x = 1/9$
- **D** $x = 1/6$

Common Mistakes to Avoid

✗ Writing $\log_2(8) = 2$ because "$\log_2$ means divide by 2".

✓ $\log_2(8) = 3$ because $2^3 = 8$. A logarithm is an exponent, not a division.

✗ Evaluating log(-5) or log(0) — treating negative/zero arguments as valid.

✓ The argument of a logarithm must be strictly positive ($x > 0$). log(-5) and log(0) are undefined.

✗ Confusing $\log_b(b) = b$ with the correct value.

✓ $\log_b(b) = 1$ always, because $b^1 = b$. Similarly, $\log_b(1) = 0$ because $b^0 = 1$.

✗ Forgetting to convert to exponential form when solving $\log_b(x) = y$ for x.

✓ Always convert first: $\log_b(x) = y \rightarrow b^y = x$. Then solve for the unknown.

Math Tips

★ "A logarithm is an exponent." Repeat this until it's automatic — it's the entire definition.

★ The two inverse properties are your fastest tools: $\log_b(b^n) = n$ and $b^{\log_b x} = x$. Memorize both.

★ To evaluate $\log_b(x)$ mentally: ask "b to what power gives x?" Write it as b^? = x and solve.

★ log(x) always means base 10 on calculators and in most textbooks. ln(x) always means base e.

★ Negative logarithms are valid — they just mean the argument is between 0 and 1. $\log_2(1/8) = -3$ because $2^{-3} = 1/8$.